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THE EFFECT OF BREAKUP AND COALESCENCE OF BUBBLES ON MASS TRANSFER IN A FLUIDIZED BED*

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The mechanics of interpenetrating and mutually reacting continua is used to study a plane problem, modelling the motion of the solid and liquid phase and mass transfer to bubbles during their breakup and coalescence. It is assumed that the interphase resistance is described, as in the Davidson model /1/, by D'arcy's Law, with the concentration of solid phase outside the bubbles remaining constant. The phase velocity field and pressure field of the liquid phase are constructed and, the conditions for the existence and form of the cloud circulation of the fluid are determined near two intersecting or mutually touching circular bubbles. A solution of the problem of diffusive material flow to the cloud surface is given and the mass transfer coefficient between the bubbles and the continuous phase of the fluidized bed is determined. The results obtained enable the influence of the breakup and coalescence of the bubbles and the presence of a rigid wall near the bubble and of bubble deformation on the intensity of the mass transfer process, to be assessed.

In the majority of cases when systems are fluidized with a gas, conditions occur which ensure the formation of moving stable cavities practically free of solid particles and resembling the bubbles forming in homogeneous liquids. The analysis of the mechanism of motion of the solid and liquid phase near such cavities and their mass transfer to the continuous phase is important in connection with the practical problem of increasing the efficiency of reactors with a fluidized bed in various physico-chemical processes. Mass transfer in the reactor can be influenced by the deformation of the bubbles during their ascent, the presence of various types of obstacles (e.g. heat exchange surfaces, rigid walls, etc.) in the working space of the reactor, and the process of breakup and coalescence of the bubbles (/2–3/ et al.). Earlier studies included that of the motion and mass transfer of a single circular (spherical) bubble /1–5/, the motion of a bubble with concave rear part /6/ and of a circular bubble between parallel walls /7/.

1. Phase flow fields around the cavity. Let us consider the simplest plane model of the motion of two touching or intersecting bubbles rising with some constant velocity U_{cav} through a homogeneous fluidized bed, and representing a cavity whose size and form does not change with time. The flow of fluidizing agent is homogeneous at large distances from the cavity, directed vertically upwards, and its velocity (in the gaps between particles) is equal to v_0 in the laboratory coordinate system.

We shall consider plane cavities whose form is described by two different circular segments (of radius a_b) constructed in a common section (Fig.1). According to experimental data (/2/, ch.4) single collapsing bubbles in fluidized systems have a form similar to the one considered here. Moreover, the motion of such a cavity can serve as the simplest model of the process of coalescence of two single bubbles and of the behaviour of a deformed (or a single circular) bubble near a rigid wall. Fig.2 shows cine frames (/2/, ch.4) depicting the breakup of a single bubble in an air-fluidized bed of glass spheres of 230 microns diameter.

In the simplest model /1/ of a fluidized bed regarded as a double continuum, the initial system of locally averaged stationary equations of motion and continuity of the liquid and solid phase identified with ideal (in linear scales of the order of the bubble size) incompressible fluids, has the following form outside the cavity (see e.g. /4/):

$$\mathbf{v}-\mathbf{w}=-k(\varepsilon)\nabla p_f,\quad d_s\rho(\mathbf{w}\nabla)\mathbf{w}=-\nabla(p_f+p_s)+d_s\rho\mathbf{g} \tag{1.1}$$
$$\nabla\mathbf{v}=\nabla\mathbf{w}=0$$

Here $\mathbf{v}$ and $\mathbf{w}$, p_f and p_s are the velocities and pressures of the fluid phase (index f) and solid phase (index s) respectively, ε is the constant porosity of the bed, $\rho=1-\varepsilon$ is

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the volume concentration of the solid particles, d_s is density, g is the acceleration due to gravity and $k(\varepsilon)$ is the permeability coefficient characterizing the filtering properties of the bed.

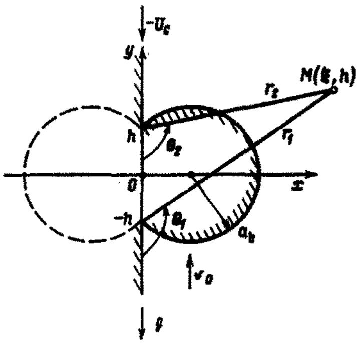

Fig.1

The motion of the fluidizing agent is assumed to be non-dispersive in the model adopted, i.e. $d_f \to 0$ (when dense beds are fluidized with gas, the inertia of the fluid is small compared with the inertia of the particles: $d_f\varepsilon \ll d_s\rho$) so that terms proportional to d_f are omitted from (1.1).

Let us consider the boundary conditions at the cavity surface and at infinity. Assuming that the fluid phase is inertialess, we find that the boundary condition at the surface S of the cavity in question is the condition that the fluid pressure is constant /1/ (p_f is measured from its value on S)

$$p_f|_S = 0 \tag{1.2}$$

At infinite distance from the moving cavity, where there are no phase flow perturbations caused by the cavity, the condition of constancy of the pressure gradient in the fluidizing agent is given in the form

$$\left.\frac{\partial p_f}{\partial y}\right|_\infty = -J = -\frac{v_0}{k} \tag{1.3}$$

Here y is the vertical coordinate (Fig.1) and J is a constant equal to the weight of unit volume of the continuous phase in the fluidized bed.

Since the field of external mass forces is conservative, the second and fourth equation of (1.1) imply the possibility of constructing a potential field of flow of the solid phase as a flow of an ideal incompressible fluid with pressure $p_f + p_s$ when $\operatorname{rot} \mathbf{w} = 0$.

After taking the divergence of both sides of the first equation of (1.1) and using the equation of phase continuity, we reduce the problem of determining the pressure distribution of the fluidizing agent outside the cavity, taking condition (1.1) into account, to the solution of the Dirichlet problem for the Laplace equation

$$\Delta p_f = 0 \tag{1.4}$$

The harmonic function p_f must satisfy the condition at infinity (1.3). Using the pressure distribution in the gas obtained in this manner, and the flow field of the solid phase, we find, using the first equation of (1.1), the velocity field of the fluidizing agent.

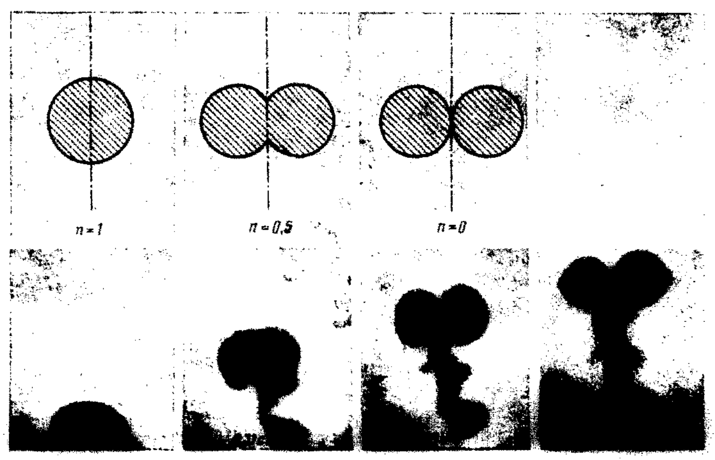

Fig.2

Henceforth we shall use in the course of the analysis the following system of dimensionless, orthogonal coaxial coordinates $\xi = \theta_1 - \theta_2$, $\eta = \ln(r_2/r_1)$, depicted in Fig.1. To construct the velocity field of the fluid phase, we write the first equation of (1.1) in dimensionless form in the system of curvilinear coordinates (ξ, η)

$$v_\xi = w_\xi - \frac{\delta}{\sqrt{g_{\xi\xi}}}\frac{\partial p_f}{\partial \xi}, \quad v_\eta = w_\eta - \frac{\delta}{\sqrt{g_{\eta\eta}}}\frac{\partial p_f}{\partial \eta} \tag{1.5}$$

Here $g_{\xi\xi}$ and $g_{\eta\eta}$ are the metric tensor components, while the characteristic values of the velocities, linear dimension and pressure are represented by the velocity of steady ascent of the cavity U_{cav}, a_b and Ja_b respectively, and $\delta = v_0/U_{cav} = kJ/U_{cav}$.

We shall also use the fact that the vector w is solenoidal and introduce the dimensionless stream function of the solid phase ψ_s so that

$$w_\xi = \sqrt{\frac{g_{\xi\xi}}{g}}\frac{\partial\psi_s}{\partial\eta} = \frac{1}{\sqrt{g_{\eta\eta}}}\frac{\partial\psi_s}{\partial\eta} \tag{1.6}$$

$$w_\eta = -\sqrt{\frac{g_{\eta\eta}}{g}}\frac{\partial\psi_s}{\partial\xi} = -\frac{1}{\sqrt{g_{\xi\xi}}}\frac{\partial\psi_s}{\partial\xi}, \quad g = g_{\xi\xi}g_{\eta\eta}$$

Now let $p_f^*(\xi, \eta)$ be the function harmonically conjugate to $p_f(\xi, \eta)$. Then by virtue of the Cauchy-Riemann conditions we have

$$\frac{1}{\sqrt{g_{\xi\xi}}}\frac{\partial p_f}{\partial\xi} = -\frac{1}{\sqrt{g_{\eta\eta}}}\frac{\partial p_f^*}{\partial\eta}, \quad \frac{1}{\sqrt{g_{\eta\eta}}}\frac{\partial p_f}{\partial\eta} = \frac{1}{\sqrt{g_{\xi\xi}}}\frac{\partial p_f^*}{\partial\xi} \tag{1.7}$$

Substituting (1.6) and (1.7) into (1.5), we obtain the following expressions for the fluid phase velocity components:

$$v_\xi = \frac{1}{\sqrt{g_{\eta\eta}}}\frac{\partial}{\partial\eta}(\psi_s + \delta p_f^*), \quad v_\eta = -\frac{1}{\sqrt{g_{\xi\xi}}}\frac{\partial}{\partial\xi}(\psi_s + \delta p_f^*)$$

from which it follows that the required velocity field of the fluidizing agent is described by the stream function $\psi_f = \psi_s + \delta p_f^*$. The function ψ_s in the last relation is determined uniquely by the complex flow potential of the solid phase given by the expression /8/

$$W_s(\zeta) = \frac{2i\gamma}{n}\operatorname{ctg}\frac{\zeta}{n} \tag{1.8}$$

$$\zeta = \xi + i\eta, \quad z = x + iy = \gamma\operatorname{ctg}\frac{\zeta}{2}, \quad \gamma = \sin\frac{\pi n}{2} = \frac{h}{a_b}$$

(x and y are rectangular Cartesian coordinates (Fig.1)). The parameter n characterizes the various forms of the cavities under consideration, $n = 0$ corresponds to a cavity in the form of two circles of equal radius in contact with each other, for $n = 1$ the cavity has the form of an isolated circle, and when $n = 2$, its boundary degenerates to a segment of the axis of symmetry of length $2h$. Fig.1 depicts the case of $n \in (0, 1)$. If on the other hand $n \in (1, 2)$, the cavity has the form of a lens.

Separating in (1.8) the imaginary part we find, that the flow of solid phase is described by the stream function

$$\psi_s = \frac{2\gamma}{n}\sin\frac{2\xi}{n}\left(\operatorname{ch}\frac{2\eta}{n} - \cos\frac{2\xi}{n}\right)^{-1} \tag{1.9}$$

which corresponds to the flow of an ideal incompressible fluid, homogeneous at infinity, past a rigid vertical wall $\xi = 0$ with an obstacle in the form of a circular segment $\xi = \pi n/2$, or of a figure symmetrical about the straight line $\xi = 0$ consisting of two circular segments. To obtain the velocity field of the fluidizing agent, it remains to find the pressure distribution p_f outside the rising cavity, having already constructed the solution of (1.4) with boundary condition (1.2) and the condition at infinity (1.3).

We know from the filtration theory that problems of this type are equivalent to determining the complex potential of a plane flow. We shall therefore determine p_f and p_f^* as follows. Let an analytic function W be found in any manner, such that

$$(\operatorname{Im} W)|_S = 0, \quad \left.\frac{dW}{dz}\right|_{z=\infty} = \left.\frac{\partial(\operatorname{Im} W)}{\partial y}\right|_{z=\infty} = -1 \tag{1.10}$$

Turning now to (1.4) and conditions (1.2) and (1.5) written in dimensionless form, we conclude by virtue of the uniqueness of the solution of the Dirichlet problem for the Laplace equation, that $p_f = \operatorname{Im} W$, $p_f^* = \operatorname{Re} W$. Obviously, the analytic function W defined by conditions (1.10) represents the analogue of the complex potential of the flow past a body of prescribed shape, of an ideal fluid, in a direction antiparallel to the x axis of a rectangular Cartesian coordinate system.

We construct the function W by conformal mapping onto the unit circle /8, 9/. The simplest case is that of $n = 0$, when the region mapped onto the z plane has the form of two unit circles touching each other. This configuration corresponds to a cavity formed by two identical circular bubbles merged at a single point, or to a single bubble near a vertical wall.

The sequence of mappings of the interior of a region in the z plane onto a unit circle in the σ plane is shown for the case $n = 0$ in Fig.3.

The mapping function is

$$\sigma(z) = \operatorname{ctg}\frac{\pi}{2z}, \quad \sigma(\infty) = \infty, \quad \left|\frac{d\sigma}{dz}\right|\Big|_{z=\infty} = \frac{2}{\pi} \tag{1.11}$$

The analogue of the complex flow potential in the σ plane has the simple form $W(\sigma) = -(\sigma + \sigma^{-1})$, and this finally yields the following expression, taking the normalizing conditions in (1.11) into account:

$$W(z) = -\frac{\pi}{2}\left(\operatorname{ctg}\frac{\pi}{2z} + \operatorname{tg}\frac{\pi}{2z}\right) \tag{1.12}$$

We generalize the latter relation to the case when $n \neq 0$ by first writing the expression for the complex flow potential of the solid phase around the cavity of the type in question. In the limit, as $n \to 0$, we obtain from (1.8)

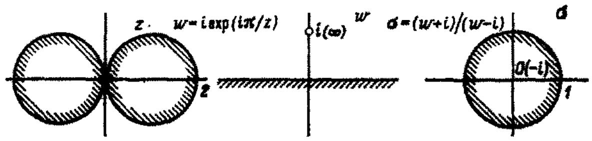

Fig.3

$$W_s(\zeta) = i\pi \operatorname{ctg}\frac{\pi}{z} \tag{1.13}$$

Comparing (1.13) and (1.12) and using the general expression (1.8) for the complex potential $W_s(\zeta)$, we construct the desired function $W(\zeta)$ for $n \neq 0$ in the form

$$W(\zeta) = -\frac{\gamma}{n}\left(\operatorname{ctg}\frac{\zeta}{2n} + \operatorname{tg}\frac{\zeta}{2n}\right), \quad z = \gamma \operatorname{ctg}\frac{\zeta}{2}, \quad 0 < n < 2 \tag{1.14}$$

Let us check whether the function constructed satisfies condition (1.10). Differentiating (1.14) we obtain

$$\frac{\partial(\operatorname{Im} W)}{\partial y} + i\frac{\partial \operatorname{Im} W)}{\partial x} = \frac{dW}{dz} = \frac{dW}{d\zeta}\frac{d\zeta}{dz} = \frac{1}{n^2}\left[\left(\sin\frac{\zeta}{2} \Big/ \cos\frac{\zeta}{2n}\right)^2 - \left(\sin\frac{\zeta}{2} \Big/ \sin\frac{\zeta}{2n}\right)^2\right]$$

Passing now to the limit as $\xi \to 0, \eta \to 0$, i.e. $\zeta \to 0$), which corresponds to moving away from the cavity $(z \to \infty)$, we obtain

$$dW/dz \xrightarrow[\substack{\zeta\to 0\\ z\to\infty}]{} -1, \text{ i.e. } \partial(\operatorname{Im} W)/\partial y \xrightarrow[\substack{\zeta\to 0\\ z\to\infty}]{} -1$$

Moreover $(\operatorname{Im} W)|_S = 0$, which can be confirmed directly from (1.14).

Separating the real and imaginary parts in (1.14), we obtain the pressure distribution and stream function of the fluidizing agent in the form

$$p_f = \operatorname{Im} W(\zeta) = \frac{4\gamma}{n}\operatorname{sh}\frac{\eta}{n}\cos\frac{\xi}{n}\left(\operatorname{ch}\frac{2\eta}{n} - \cos\frac{2\xi}{n}\right)^{-1} \tag{1.15}$$

$$\psi_f = \psi_s + \delta \operatorname{Re} W(\zeta) = \tag{1.16}$$

$$-\frac{4\gamma}{n}\sin\frac{\xi}{n}\left(\delta \operatorname{ch}\frac{\eta}{n} - \cos\frac{\xi}{n}\right)\left(\operatorname{ch}\frac{2\eta}{n} - \cos\frac{2\xi}{n}\right)^{-1}$$

2. Regions of closed circulation and modes of fluid phase flow. From (1.16) it follows that in the case when the velocity of steady ascent of the cavity U_{cav} exceeds the velocity of fluidization v_{0}, i.e. $\delta = v_0/U_{cav} < 1$, a region of closed circulation of the fluidizing agent (a cloud) appears near the cavity. The cloud boundary is described by the equation

$$\delta \operatorname{ch}\frac{\eta}{n} - \cos\frac{\xi}{n} = 0 \tag{2.1}$$

When $\delta > 1$, Eq.(2.1) has no real solutions. In this case no closed stream lines form in the fluid phase near the cavity, i.e. we have a continuous-type flow. Fig.4 depicts the evolution of the cloud boundary as a function of δ for the case when $n = 0$.

Henceforth, we consider the case of a circulation mode of the liquid phase flow $(U_{cav} > v_0)$ in which the ascending cavity is surrounded by a cloud. The gas trapped within the cloud circulates with great intensity, so that it is natural to assume complete mixing in the fluid phase and the constancy of the reagent concentration within the cloud.

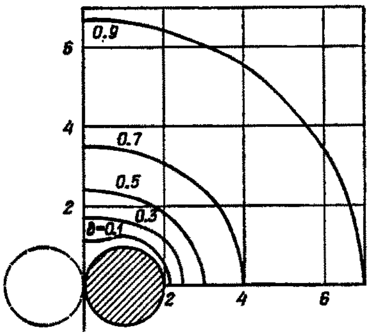

Fig.4

3. Mass transfer between the cavity and the continuous phase. We shall consider the case when the Peclet number defined in terms of the characteristic size of the region of closed circulation l, the velocity $U_{cav}-v_0$ of motion of the cavity relative to the homogeneous flux of the fluid phase and the effective diffusion coefficient D, is large compared with unity $\mathrm{Pe} = l\,(U_{cav} - v_0)/D \gg 1$. In this case the resistance to mass transfer towards the cavity is centered, near the cloud boundary, in the region of the diffusive boundary layer (/10/ et al.). The problem of mass transfer between the cavity and the continuous phase of the fluidized bed is thus reduced to computing the diffusive flux towards the cloud boundary /5/.

The steady concentration field of the reacting species in the region outside the cloud is described by the equation of steady convective diffusion. We shall write this equation, together with the condition reflecting the perturbation-free state of concentration away from the cavity and the complete absorption of the dissolved material at the cloud boundary, in the following, dimensionless standard form:

$$\frac{1}{\sqrt{g_{\nu\nu}}} v_\nu \frac{\partial c}{\partial \nu} + \frac{1}{\sqrt{g_{\varkappa\varkappa}}} v_\varkappa \frac{\partial c}{\partial \varkappa} = \frac{\mathrm{Pe}^{-1}}{\sqrt{g}} \left[\frac{\partial}{\partial \nu} \left(\frac{\sqrt{g}}{g_{\nu\nu}} \frac{\partial c}{\partial \nu} \right) + \frac{\partial}{\partial \varkappa} \left(\frac{\sqrt{g}}{g_{\varkappa\varkappa}} \frac{\partial c}{\partial \varkappa} \right) \right] \tag{3.1}$$

$$c\,|_{\nu\to\infty} \to 1, \quad c\,|_{S_c} = 0$$

Here ν and $\varkappa$ denote the normal and tangential coordinates attached to the surface S_c bounding and cloud, whose equation in the $(\nu, \varkappa)$ coordinate system has the form $\nu = \nu_0$; $g_{\nu\nu}$, $g_{\varkappa\varkappa}$ are the components of the metric tensor $g = g_{\nu\nu} g_{\varkappa\varkappa}$. The concentration c is counted from its value within the cloud with closed circulation, and is referred to the value of the concentration away from the cavity.

We note that the condition of quasistationarity of the diffusion process within the diffusive boundary layer is $T \gg t_D$ where T is the characteristic time of the bubble remaining within the bed: $T = L/U_{cav}$ (L is the linear dimension of the reactor) and t_D is the characteristic diffusion time (the time taken for the diffusive boundary layer to become established): $t_D = \delta_D^2/D = l^2 D^{-1} \mathrm{Pe}^{-1}$ (δ_D is the thickness of the diffusion boundary layer $\delta_D \sim l\,\mathrm{Pe}^{-1/2}$).

We use the method of matching the asymptotic expansions to obtain approximate solution of (3.1). The method is based on separating from the flow space regions within which the flow can be simplified after equating the orders of separate terms of the equation. The solutions of the simplified equation (3.1) are obtained for every single region, and become matched at their boundaries.

Following the method, we introduce in the region of diffusion boundary layer a stretched normal coordinate Y according to the formula

$$Y = \mathrm{Pe}^{1/2} (\nu - \nu_0), \quad Y = O(1) \tag{3.2}$$

The region of diffusion boundary layer is described in the $(Y, \varkappa)$ coordinates by the relations

$$\nu - \nu_0 \leqslant \mathrm{Pe}^{-1/2} Y, \quad \varkappa^- \leqslant \varkappa \leqslant \varkappa^+ - O(\mathrm{Pe}^{-1/2}) \tag{3.3}$$

where the coordinates $\varkappa^-$ and $\varkappa^+$ correspond to the leading and trailing stagnation points. In the region of trailing stagnation point where $\nu - \nu_0 \leqslant \mathrm{Pe}^{-1/2} Y$, $\varkappa^+ - O(\mathrm{Pe}^{-1/2}) \leqslant \varkappa \leqslant \varkappa^+$, the diffusion boundary layer approximation is not suitable /10/. The contribution of this region to the total flux of material to the surface $\nu = \nu_0$ is, however small, compared with that of the region of the diffusion boundary layer /10/, and is therefore disregarded.

Estimation of the order of magnitude of the terms of (3.1) in the region (3.3) is based on expanding the stream function of the fluidizing agent in a series in powers of $\nu - \nu_0$ near the cloud boundary. The expansion has the form

$$\psi_f = (\nu - \nu_0) f(\varkappa) + O[(\nu - \nu_0)^2], \quad f(\varkappa) = \left. \frac{\partial \psi_f(\nu, \varkappa)}{\partial \nu} \right|_{\nu=\nu_0} = -\left(\sqrt{g_{\nu\nu}}\, v_\varkappa\right)|_{\nu=\nu_0} \tag{3.4}$$

Using expressions (3.3) and (3.4) we can reduce (3.1), after discarding terms of higher order of smallness in the parameter $\mathrm{Pe}^{-1/2}$, to the equation of heat conduction * (See: Gupalo Yu.P., Polyanin A.D. and Ryazantsev Yu.S. Mass transfer between a bubble drop and laminar fluid flow at large Peclet numbers. Preprint In-ta prikl. mekhaniki Akad. Nauk SSSR, Moscow, No.120.)

$$\partial c/\partial \tau = \partial^2 c/\partial z^2 \tag{3.5}$$

Here Z is the Mises varible $Z = Yf(\varkappa)$, and the new variable τ is connected with the coordinate $\varkappa$ by the relation

$$\tau(\varkappa, \varkappa^-) = \left| \int_{\varkappa^-}^{\varkappa} v_\varkappa^\circ(\lambda) \sqrt{\overset{\circ}{g}_{\varkappa\varkappa}(\lambda)}\, d\lambda \right| \tag{3.6}$$

where the superscript "o" means that the quantity in question is taken at the surface $\nu = \nu_0$.

The boundary conditions for (3.5) are

$$Z \to \infty,\ c \to 1;\ Z = 0,\ c = 0;\ Z \neq 0,\ \tau = 0,\ c = 1 \tag{3.7}$$

(the first and third condition follows from the condition of matching with the unperturbed concentration field in the outer region, and the second condition follows from the second condition of (3.1)).

The solution of problem (3.5), (3.7) yields the concentration distribution in the region of the diffusion boundary layer, and has the form /11/

$$c(Z, \tau) = \operatorname{erf}\left(\frac{Z}{2\sqrt{\tau}}\right) \tag{3.8}$$

Differentiating (3.8) along the normal to the cloud surface, we obtain an expression for the local dimensionless diffusion flux onto this surface

$$j(\varkappa) = \frac{1}{\sqrt{\overset{\circ}{g}_{\nu\nu}(\varkappa)}} \left(\frac{\partial c}{\partial \nu}\right)^\circ = \frac{\mathrm{Pe}^{1/2} |v_\varkappa^\circ(\varkappa)|}{\sqrt{\pi\tau(\varkappa, \varkappa^-)}}$$

The total material flux to the boundary of the region of closed circulation in the two-dimensional problem, is then equal to

$$I = \int_{S_c} j(\varkappa)\, d\varkappa = 2 \int_{\varkappa^+}^{\varkappa^-} \frac{\mathrm{Pe}^{1/2}\, v_\varkappa^\circ(\lambda)}{\sqrt{\pi\tau(\lambda)}} \sqrt{\overset{\circ}{g}_{\varkappa\varkappa}(\lambda)}\, d\lambda = \frac{4\,\mathrm{Pe}^{1/2}}{\sqrt{\pi}} \sqrt{\tau(\varkappa^-, \varkappa^+)} \tag{3.9}$$

Here S_c is the cloud surface defined by (2.1). Let us introduce on it a natural metric by putting $\overset{\circ}{g}_{\varkappa\varkappa} = 1$. In this case the coordinate $\varkappa$ represents simply the cloud boundary arc length. Calculating the value of $\tau(\varkappa^-, \varkappa^+)$ in (3.9), we pass to coaxial coordinates (ξ, η) introduced above, remembering that

$$d\varkappa^2 = \overset{\circ}{g}_{\xi\xi}\, d\xi^2(\eta) + \overset{\circ}{g}_{\eta\eta}\, d\eta^2 = \overset{\circ}{g}_{\eta\eta}\left[1 + \frac{\overset{\circ}{g}_{\xi\xi}}{\overset{\circ}{g}_{\eta\eta}} \left(\frac{d\xi}{d\eta}\right)^{02}\right] d\eta^2 \tag{3.10}$$

Using the coordinates (ξ, η) we obtain the equation of the cloud boundary in the form $\xi = \xi(\eta)$ directly from (2.1).

Using (3.10) we transform (3.6) for the variable τ to the form

$$\tau(\eta, \eta^-) = \left| \int_{\eta^-}^{\eta} v_\varkappa^\circ(\eta') \left(\frac{d\varkappa}{d\eta'}\right)^\circ d\eta' \right| = \int_{\eta^-}^{\eta} v_\varkappa^\circ(\eta') \sqrt{\overset{\circ}{g}_{\eta\eta}(\eta')} \left[1 + \left(\frac{d\xi}{d\eta'}\right)^{02}\right]^{1/2} d\eta', \quad \eta = -n \operatorname{arch} \delta^{-1} \tag{3.11}$$

since in coaxial coordinates $g_{\xi\xi} = g_{\eta\eta}$.

We will reduce the stream function of the fluidizing agent obtained in Sect.1 to dimensionless form, by introducing the quantity $U_{\mathrm{cav}} - v_0$ as the characteristic velocity and the characteristic size of the cloud l as the length. From (1.16) we obtain

$$\psi_f = -\frac{a_b}{1-\gamma} \frac{4\gamma}{ln} \sin\frac{\xi}{n} \left(\gamma \operatorname{ch}\frac{\eta}{n} - \cos\frac{\xi}{n}\right) \left(\operatorname{ch}\frac{2\eta}{n} - \cos\frac{2\xi}{n}\right)^{-1}$$

From this, remembering that $g_{\xi\xi} = g_{\eta\eta} = a_b^2\gamma^2/(\operatorname{ch}\eta - \cos\xi)^2 l^2$, $v_\varkappa^\circ = (v_\xi^{\circ 2} + v_\eta^{\circ 2})^{1/2}$, and taking into account the expressions for the fluid phase analogous to (1.6), we arrive at the final formula for the integral flux

$$I = \frac{8\mathrm{Pe}^{1/2}}{\sqrt{\pi}} \left(\frac{\gamma}{n}\right)^{1/2} \left(\frac{1+\delta}{1-\delta}\right)^{1/4} \left(\frac{a_b}{l}\right)^{1/2} \tag{3.12}$$

The mean Sherwood number obtained from (3.12) is (β is the mass transfer coefficient)

$$\mathrm{Sh} = \frac{l\beta}{D} = \frac{I}{2\pi} = \frac{4\mathrm{Pe}^{1/2}}{\pi^{3/2}} \left(\frac{\gamma}{n}\right)^{1/2} \left(\frac{1+\delta}{1-\delta}\right)^{1/4} \left(\frac{a_b}{l}\right)^{1/2} = 4\left(\frac{1}{n}\sin\frac{\pi n}{2}\right)^{1/2} \frac{(1-\delta^2)^{1/4}}{\pi^{3/2}} \left(\frac{a_b U_{\mathrm{cav}}}{D}\right)^{1/2} \tag{3.13}$$

Let us consider a number of special cases corresponding to particular forms of the ascending cavity.

A single circular bubble of radius a_b. In this case the expression for the mean Sherwood number follows from (3.13) with $n = 1$ and $a_b/l = ((1-\delta)/(1+\delta))^{1/2}$ (the cloud boundary is a circle of radius $((1+\delta)/(1-\delta))\,a_b$)

$$\mathrm{Sh} = \frac{4}{\pi^{3/4}}\,\mathrm{Pe}^{1/2} = \frac{4(1-\delta^2)}{\pi^{3/4}}\left(\frac{a_b U_{\rm cav}}{D}\right)^{1/2} \tag{3.14}$$

which agrees with the result obtained earlier in /5/.

Two equal circular bubbles of radius a_b *touching each other.* Passing in (3.13) and (2.13) to the limit as $n \to 0$, we obtain the following expression for the mean Sherwood number:

$$\mathrm{Sh} = \frac{2\sqrt{2}}{\pi}(1-\delta^2)^{1/4}\left(\frac{a_b U_{\rm cav}}{D}\right)^{1/2} \tag{3.15}$$

A single circular bubble of radius a_b *rising along a vertical rigid wall.* Here the result (3.15) for the mean Sherwood number is halved.

We recall the relations (3.12)–(3.15) were obtained under the assumption that $\mathrm{Pe} = l(U_{\rm cav} - v_0)/D \gg 1$. For the slowly ascending cavities, when $U_{\rm cav} \to v_0$, the formulas shown hold provided that $1-\delta \gg Dn/(2h/U_{\rm cav})$ (here the half-width of the cloud $l = h/\mathrm{tg}\,(^1/_2 n \cdot \arccos\delta)$ is taken as its characteristic dimension).

Relations (3.13)–(3.15) enable us to assess the variation in the total material flux at the cloud surface relative to the deformation of the initial circular cavity. We shall assume, as is often done in engineering calculations /1–3/, that the ascent rate of the cavity depends only on its volume (area) and not on its form, so that $U_{\rm cav} = U_{\rm cav}(\Sigma)$. Rewriting (3.13) and (3.14) in terms of the areas Σ of the corresponding figures, we obtain

$$\mathrm{Sh} = \left(\frac{2^{7/2}U_{\rm cav}}{\pi^3 Dn}\sin\frac{\pi n}{2}\right)^{1/2}\left(\pi - \frac{\pi n}{2} + \frac{\sin \pi n}{2}\right)^{-1/4}(1-\delta^2)^{1/4}\,\Sigma^{1/4} \tag{3.16}$$

$$\mathrm{Sh} = \left(\frac{16 U_{\rm cav}}{\pi^{7/2} D}\right)^{1/2}(1-\delta^2)^{1/4}\,\Sigma^{1/4} \tag{3.17}$$

We shall also assume that the cavity volume is not affected by its deformation ($\Sigma = \mathrm{const}$). Dividing (3.16) by (3.17), we arrive at the following expression for the ratio of the total material fluxes to the cloud in the course of the deformation of the circular cavity:

$$F(n) = 2^{-1/4}\left(n^{-1}\sin\frac{\pi n}{2}\right)^{1/2}\left(1 - \frac{n}{2} + \frac{\sin \pi n}{2\pi}\right)^{-1/4} \tag{3.18}$$

Analysis of relation (3.18) depicted in Fig.5 shows that deformations of the cavity lead, in general, to an increase in the intensity of mass transfer to the continuous phase. The increase in the material flux to the cloud is small for all values of n, except at $n = 2$. Thus, if a single circular bubble is deformed into a double bubble ($n = 0$, see Fig.2), the increase is of the order of 5%.

A more accurate assessment of the variation in the diffusive material flux to the cloud surface when the bubble is deformed, can be carried out provided that the relation connecting the coefficient of resistance and the middle cross-section of the ascending cavity with its form, is known.

Writing in the usual form the balance of dynamic pressure in the flow impinging on the deformed bubble and the force propelling it upwards, we arrive at the following expression for the velocity of ascent of a two-dimensional cavity:

$$U_{\rm cav} = K l_{\rm mid}^{-1/2}, \quad K = (2g\Sigma/c_x)^{1/2} \tag{3.19}$$

Here c_x is the heat resistance coefficient and $l_{\rm mid}$ is the length of the middle cross-section of the cavity.

Formula (3.19) allows us, unlike the earlier assumption that $U_{\rm cav} = U_{\rm cav}(\Sigma)$, to take into account the dependence of the variation in the velocity of cavity ascent on its form.

The ratio of integral fluxes at the cloud surface after and before the deformation can be written using (3.16), (3.17) and (3.19) in the form

$$F_1(n, c_x, v_0) = \left[\sqrt{2}\left(1 - \frac{n}{2} + \frac{\sin \pi n}{2\pi}\right)^{1/2} \times \left(1 + \cos\frac{\pi n}{2}\right)^{-1}\right]^{1/4} F(n)\,\frac{1 - v_0^2/U_{\rm cav}^2}{1 - v_0^2/U_b} \tag{3.20}$$

Here the expression within the square brackets represents the ratio of the bubble diameter to the length of the middle cross-section after the deformation, the function $F(n)$ is given by (3.18) and U_b is the rate of ascent of the bubble.

When the cavity volume remains unchanged under a deformation of the form discussed here, the following expression for the length of its middle cross-section holds:

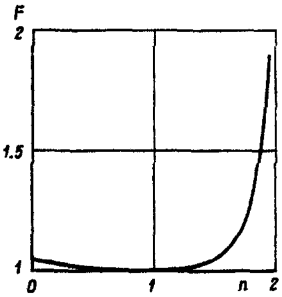

Fig.5

$$l_{\rm mid} = \sqrt{2}\left(1+\cos\frac{\pi n}{2}\right)\left(1-\frac{n}{2}+\frac{\sin \pi n}{2\pi}\right)^{-1/2} a_b$$

where a_b is the bubble radius prior to deformation. Without the deformation we have $(n = 1)\ l_{\rm mid} = 2a_b$, when a double bubble forms we have $(n = 0)\ l_{\rm mid} = 2\sqrt{2}a_b$, and when the bubble stretches along the stream, we have $(n \to 2)\ l_{\rm mid} \to 0$.

It follows from (3.20), in particular, that when the bubble is deformed along the flow $(1 < n < 2,\ l_{\rm mid} < 2a_b,\ U_{\rm cav} > U_b)$, the mass transfer process is intensified compared with (3.18), obtained when the form of the bubble in unchanged, since the increase in the integral material flux to the cloud is related not only to the increase in its surface area, but also to the increase in the rate of ascent of the cavity. Conversely, when the deformations take place in a direction transverse to the flow $(0 < n < 1,\ l_{\rm mid} > 2a_b,\ U_{\rm cav} < U_b)$, the increase in the cloud area is compensated by the slower ascent of the bubble, so that the material flux to the cloud may, in effect, be reduced.

The results obtained enable us to estimate the variation in the mass transfer coefficient in processes such as coalescence of two or more single circular bubbles, splitting of a single circular bubble and motion of a bubble along a rigid vertical wall.

Thus, in the first case it follows from (3.17) that when a large number m of identical circular bubbles infinitely distant from each other at the initial instant, coalesce, the mass transfer intensity is reduced in accordance with the formula

$$m\,{\rm Sh}/{\rm Sh}_0 = m^{1/4}(1-\delta^2)^{3/4}(1-\delta^2 m^{-1/2})^{-1/4} \sim m^{1/4} > 1$$

for large m. The Sherwood number Sh corresponds here to a single circular bubble, the quantity δ is found from its rate of ascent, and the quantity ${\rm Sh}_0$ characterizes the mass transfer with the continuous phase of the bubble resulting from the coalescence.

Conversely, when breakup occurs the mass transfer is intensified

$${\rm Sh}_0/(m\,{\rm Sh}) \sim m^{-1/4} < 1,\quad m \gg 1$$

which is explained by the increase (after breakup) in the area of contact of the "total cloud" with the continuous phase. Clearly, the mass transfer will increase still more, provided that the breakup produces bubbles small enough to become non-circulating, $(\delta > 1)$.

Relations (3.14) and (3.15) also imply that in the case of a circular bubble moving along a rigid wall, the material flux towards the cloud surface will be reduced by almost 40% because of the wall screening effect, compared with the case of identical size bubbles ascending at the same rate through an unbounded medium (the ratio of the integral fluxes is $\sqrt{\pi/8} \cong 0.627$).

REFERENCES

1. DAVIDSON J.F. and HARRISON D., Fluidized Particles. Cambridge University Press, 1963.
2. DAVIDSON J.F. and HARRISON D., Fluidization. Academic Press, London, N.Y., 1971.
3. GEL'PERIN N.I., EINSTEIN V.G. and KVASHA V.B., Fluidization Technique Fundamentals, Moscow, Khimia, 1967.
4. BORODULYA V.A. and GUPALO YU.P., Mathematical Models of Chemical Reactors with Fluidized Beds. Minsk, Nauka i tekhnika, 1976.
5. GUPALO YU.P., RYAZANTSEV YU.S. and SERGEEV YU.A., On the mass transfer between bubbles and continuous phase in a fluidized bed. Izv. Akad. Nauk SSSR, MZhg, No.4, 1973.
6. MURRAY J.D., On the mathematics of fluidization. Pt. II. Steady motion of fully developed bubbles. J. Fluid Mech., Vol.22, pt.1, 1965.
7. COLLINS R., An extension of Davidson's theory of bubbles in fluidized beds. Chem. Engng Sci., Vol.20, No.8, 1965.
8. MILNE-THOMSON L.M., Theoretical Hydrodynamics. London, Macmillan, 1960.
9. LAVRENT'EV M.A. and SHABAT B.V., Problems of Hydrodynamics and Their Mathematical Models. Moscow, Nauka, 1977.
10. LEVICH V.G., Physico-chemical Hydrodynamics. Moscow, Fizmatigiz, 1959.
11. TIKHONOV A.N. and SAMARSKII A.A., The Equations of Mathematical Physics. Moscow, Nauka, 1972.

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